Adaptive non-intrusive approximation of parametric PDEs with lognormal coefficients

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ABSTRACT

The diffusion coefficient of the Darcy problem with lognormally distributed random parameters can be represented as the exponential of an affine function that depends on a large number of stochastic variables.

In a previous work, a stochastic Galerkin finite element method was used to derive a fully adaptive algorithm that generates an optimal approximation of the solution with respect to the number of finite element and stochastic degrees of freedom [1]. However, the stochastic Galerkin approach is an intrusive method, which heavily relies on the specific structure of the underlying problem and is thus by nature not very adaptable.

In a different approach we utilize a non-intrusive, sample-based least-squares minimization algorithm [2] to reconstruct both the lognormal diffusion coefficient and the solution of the Darcy equation in a hierarchical tensor format.

Since the naive approach to reconstruct the lognormal diffusion coefficient via samples fails, we present a more general, efficient and precise approach that is also easily adaptable to a broad class of other function reconstruction problems. Our approach uses the better behaved reconstruction of the affine exponent and computes the exponential thereof by solving an ODE via Galerkin projection. We show that solving the resulting system is equivalent to the minimization of the approximation error. The resulting algorithm yields an approximation improvement of several orders of magnitude while being faster then the naive sample-based reconstruction algorithm.

REFERENCES
