Robust Coarse Problems for Linear and Nonlinear Domain Decomposition Methods

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ABSTRACT

Linear and nonlinear domain decomposition methods (DDMs) are robust and highly parallel scalable solution methods for discretized partial differential equations. The convergence rate of linear domain decomposition methods is generally determined by the eigenvalues of the preconditioned system. For second-order elliptic partial differential equations, coefficient discontinuities with a large contrast can lead to a deterioration of the convergence rate. Only by implementing an appropriate coarse space or second level, a robust domain decomposition method can be obtained. In this talk, a frugal coarse space for FETI-DP (Finite Element Tearing and Interconnecting - Dual Primal) and BDDC (Balancing Domain Decomposition by Constraints) methods is presented, which has a lower set-up cost than competing adaptive coarse spaces and is robust for a more general class of problems than classical FETI-DP and BDDC coarse spaces. Furthermore, a reduction of the size of the frugal coarse space is discussed. Also inexact BDDC variants based on an approximation of the frugal coarse problem are presented, using either a vertex-based approach or an AMG (algebraic multigrid) preconditioner. The frugal coarse space is compared to adaptive coarse spaces as well as classical coarse spaces, and parallel scalability up to 262 144 parallel cores for a parallel BDDC implementation is shown.

Transferring the idea of adaptive or frugal coarse spaces to the nonlinear case, there is some additional potential to save computing time. In a Newton-Krylov-DDM approach, adaptive or frugal coarse spaces can be recycled and used in several consecutive Newton iterations. In modern nonlinear FETI-DP methods, even the nonlinear convergence can be improved due to the nonlinear characteristics of the coarse problem. In this talk, some preliminary results showing some of these effects are presented.