A Space-Time FE Level-set method for convection coupled phase-change processes

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Abstract

Phase transition processes have great relevance for both engineering and scientific applications. In production engineering, for instance, metal welding and alloy solidification are topics of ongoing research. In this contribution we focus on the convection coupled solid-liquid phase change of a single species, e.g. water. The material is assumed to be incompressible within the two phases, but we account for density changes across the phase interface. To describe the process, we need to solve the incompressible Navier-Stokes equations and the heat equation for both phases over time. The position of the phase interface is tracked with a Level-set method. The Level-set function is advected according to the propagation speed of the phase interface. Such velocity field depends on local energy conservation across the interface and is modelled as the Stefan condition [1]. This formulation requires us to approximate the heat flux discontinuity across the interface based on the evolving temperature and velocity fields.

To model the temperature and velocity fields within each phase, we employ the Space-Time Finite Element method. However, commonly used interpolation functions, such as piecewise linear functions, fail to capture discontinuous derivatives over one element that are needed to assess the Level-set’s transport term. Available solutions to this matter, such as local enrichment with Extended Finite Elements, are often not compatible with existing Space-Time Finite Element codes and require extensive implementation work. Instead, we consider a conceptually simpler method and we decide to extend the Ghost Cell technique to Finite Element meshes [2]. The idea is that we can separate the two subdomains associated with each phase and solve two independent temperature problems. We prescribe the melting temperature at an additional node close to the interface and we retrieve the required heat flux.

In this work we describe the Ghost Cell method applied to our Space-Time Finite Element solver [3]. First, we verify numerical results against analytical solutions, then we demonstrate more complex test cases in 2D and 3D.

References

