Stabilized Methods for a Log-Conformation Formulation for Viscoelastic Fluids

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ABSTRACT

The microstructure of flowing media in different applications such as blood handling devices or extrusion of polymer materials is often connected to viscoelastic behaviour. In order to model these flows, the constitutive equation for Newtonian fluid flows is supplemented by an additional unknown for the deviatoric stresses. These stresses follow highly non-linear hyperbolic laws posing several different challenges for a finite element analysis.

In particular highly elastic flows are known to suffer from the so-called “High Weissenberg number problem”, which describes a breakdown of either the non-linear iteration or the linear solver at a critical problem dependent Weissenberg number. It was shown that the conformation (an intrinsic quantity of the constitutive equation) has to be positive definite and that the loss of this property is connected to a breakdown in a computation. Recent developments in the 21st century have produced so-called logarithmic conformation formulations, in which the original constitutive equation is replaced by an equivalent representation to ensure the positive definiteness of the conformation tensor by construction. Computations of a wider range of Weissenberg numbers became possible.

The present work is based on a particular formulation proposed by [1] to represent the differential Oldroyd-B model. A non-singular behaviour at zero Weissenberg number and analytic derivatives of the surrogate constitutive equation enable the use of continuation techniques starting at the Newtonian case for cost-efficient steady state computations and allow the Newton-Raphson non-linear iteration for fast convergence.

We develop common stabilized methods (for example Galerkin/Least-squares [2]) for the said formulation to address the remaining sources of numerical instabilities. These include instabilities coming from the convective nature of the momentum equation or constitutive equation, and from the choice for the interpolation spaces for velocity, pressure, and extra stresses.

We compare the accuracy and robustness of the methods by means of different benchmark problems. The design of stabilization parameters is further explored.

REFERENCES
