A space-time adaptive method for parabolic evolution equations

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ABSTRACT

Taking the well-posed mixed simultaneous space-time variational formulation introduced in [And13], we use methods previously developed in [SW20] to construct an adaptive solver for parabolic evolution equations [SvVW21]. If the solution contains singularities, then adaptivity offers a way to improve the poor convergence rates found by uniform refinement.

Exploiting the product structure of the space-time cylinder, the family of trial spaces that we consider are given as the spans of wavelets-in-time and (locally refined) finite element spaces-in-space. Restriction to sparse tensor product approximations allows to solve the whole time evolution at a complexity of solving the corresponding stationary problem.

By properties of the temporal wavelets, we can shown quasi-optimality of the discrete solutions. Combined with multigrid in space, we construct optimal preconditioners that allow for fast solution of the discrete problems. A hierarchical error estimator allows for local refinement, yielding an adaptive procedure. We show $r$-linear convergence of the approximations. By a careful implementation, we calculate the approximations in an optimal linear complexity in the number of degrees of freedom [vVW20].

We verify our theory with numerical results for the heat equation, and observe optimal convergence also for singular solutions.

REFERENCES


