An adaptive-stabilized finite element method for unsteady problems

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Key Words: Stabilized finite element, unsteady problems, parabolic, adaptive mesh refinement, advection, advection-diffusion

ABSTRACT

We present our new method of solving time-dependent problems which provides a stable solution as well as adaptivity. Considering a partial differential equation with the first derivative in time, we adopt the method of lines to discretize the temporal domain. For this, we use the generalized-\(\alpha\) method \cite{4} or high-order versions of the scheme \cite{1, 2} to deal with the derivative in time. Then, we choose a finite-dimensional space \(V_h\) composed of broken polynomials defined on the discrete spatial domain. Next, we adopt a well-posed discontinuous Galerkin variational formulation for our problem. Finally, following the idea introduced in \cite{3}, we obtain a residual-minimization problem with \(H^1\)-conforming trial space which is subset of the broken \(H^1\)-conforming test space \(V_h\). The resulting system represents a saddle-point problem and delivers a stabilized discrete solution and an error representation that drives the adaptive mesh refinement. For this, we also introduce a new discrete time-dependent norm which provides stability for our bilinear. Furthermore, we show the performance of our method by presenting some examples of dynamic advection-diffusion (parabolic) and pure advection (first-order hyperbolic).

REFERENCES


